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APPROXIMATION OF TEMPERATURE DEPENDENCE OF THE MODULUS OF ELASTICITY OF GLASS

A. I. Shutov[1] and I. V. Lakhmetkin[1]

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A refined nonlinear value of the main parameter of a material, i.e., the elongation modulus versus the instant temperature value, was suggested for introduction into the computational algorithm of tempering stresses.

The problem of analytical prediction of the properties of tempered thin (3 – 5 mm) glass is widely known. The existing methods for analytical studies of the tempering process can be referred to the following groups: the tempering theory by G. N. Bartenev [1] and the theory of O. V. Mazurin [2], which is an improvement of the algorithm developed by O. Narayanaswami together with R. Gardon [3].

The Bartenev theory developed at the end of 1940s corresponded to the engineering and technology level of that period and adequately described the process of tempering of thick glass (6 mm and more) with a low cooling intensity (the factor of heat efficiency to the cooling agent < 200 $W/(m^2 \cdot K)$).

The relaxation theory by Mazurin – Narayanaswami shows good results within a wide technological range for finite values of tempering stresses, but due to the nonlinearity of the physical processes occurring in vitrification it suffers from significant errors for current (temporary) stresses.

In the present paper it is proposed to use a more precise nonlinear expression for the main property of a material, i.e., the Young's modulus $E(t)$, where t is the instant temperature, in the computational algorithm.

Researchers of the Belgorod State Technological Academy of Construction Materials in [4] used the corrected algorithm of calculation of tempering stresses taking into account the nonlinearity of the physical constants and obtained promising results. However, it should be noted that the work was not sufficiently correct, which diminished the practical value of the method. In particular, the approximation of the experimental curve was performed with a third-degree polynomial which, first, does not ensure good mean quadratic approximation of the experimental data and, second, does not account for the physical meaning of the vitrification process.

In earlier papers, several attempts were made to approximate the elasticity modulus based on the physical essence of the vitrification process. Thus, an attempt to determine the elasticity modulus based on experimental data [5] and the verification performed according to McGraw [6] yielded more than satisfactory results, but only within a temperature range of 600 – 660°C.

The results of another experimental investigation of the elasticity modulus within a wide temperature range using various raw materials [7] were rather preliminary. Although the obtained equations had a physical meaning, they did not ensure sufficient accuracy of the calculations. In order to improve the calculation precision, some correction factors were introduced, which provided for better agreement of the rated and the experimental data, although only within certain temperature ranges. As a whole, due to the large mean quadratic deviation, the results could not be used for practical purposes.

In studying this problem, we arrived at the conclusion that the elasticity modulus should be approximated with the use of numerical methods taking into account the physical meaning of the vitrification process.

In our work we used the previously developed database of approximating functions. Proceeding from the physical meaning of the process, we chose the exponential dependence as the basic approximating function. Taking into account the Chebyshev numerical approximation [8], let us represent the space of the function $E(t) = C[a, b]$ as a line segment $[a, b]$ with the norm

$$p(f) = \sup_{x \in B} |f(x)|,$$

where B is a certain subset of the segment $[a, b]$.

[1] Belgorod State Technological Academy of Construction Materials, Belgorod, Russia.

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Using the Remez algorithm, i.e., starting with the subset

$$M^{(0)} = \left\{x_i^{(0)}\right\}_{j=0}^{n+1}, \quad a \le x_0^{(0)} < x_1^{(0)} < \ldots < x_{n+1}^{(0)} \le b,$$

as the initial approximation of the Chebyshev alternance and according to its property, we determine the respective approximation $\tilde{P}_n^{(0)}$ and the residual. By replacing a single element or all elements in the set $M^{(0)}$, we achieve a decrease of the residual, and thus arrive at a new set $M^{(1)}$, to which we apply the same procedure, etc. The process is completed when the imposed precision limit associated with the residual decrease is attained. As a result, the following approximating function was obtained.

$$E(t) = a + b \Big/ \left(1 + \exp\left(-\frac{t+c}{d}\right)\right)^e,$$

where a, b, d, and e are coefficients of the approximation; $a = 62.256482$, $b = -60.45922$, $c = 558.69427$, $d = 14.41652$, $e = 0.17988213$.

With these coefficients, the mean quadratic error was equal to $r^2 = 0.9973$.

This function is presented graphically in Fig. 1.

The proposed method can be used in the computation of deformation processes in glass articles of various types, as well as for analytical prediction of tempered glass properties.

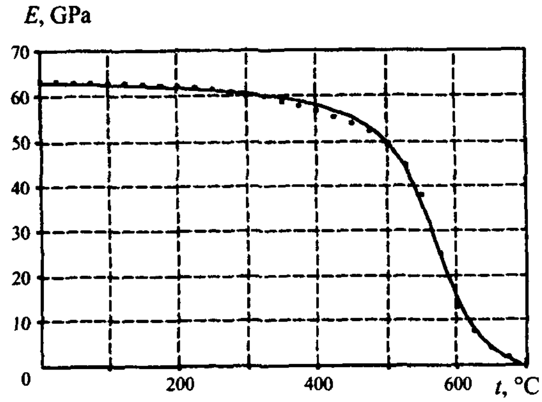

Fig. 1. Variation in elongation modulus versus glass temperature. Dots) McGraw curve; solid line) results obtained with the use of the proposed method.

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